Explaining the profit differential between two firms

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Abstract

A rather simple question in economics and management is why one firm makes more profit than another. However the answer remains unclear because of the large range of factors that might explain differential profit. In this paper we propose a decomposition of the profit differential between two firms by considering separately volume and price effects. On the volume side, we identify factors related to differences in management efficiency (technical and allocative), dominance of one firm technology over the other (productivity gaps) and size differences. On the price side, price differences among inputs and outputs prices explained by competitive advantages can also explain a big part of the profit gap. Our analysis encompasses a spatial framework where profits of two different firms are compared or a time framework by comparing profits of the same firm at two different dates. We first motivate the decomposition from a theoretical point of view and we next propose a practical and operational measure of each component based on Bennet indices. Profit differences are value measures (in money) which can be split into price and volume effects by economic indices. Individual component of the decomposition are analyzed and computed independently in a bottom-up approach before we finally show that the sum of all components allows recovering the full profit differential between two firms.

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Introduction

On the premise that profit maximization is the economic behavior of firms, we are interested in the relative performance of two firms i.e. the differential of their respective profit. At a first glance, why one firm makes more profit than another seems a rather simple question in economics and management. However a large range of factors might explain profit gaps. In this paper we are interested in the decomposition and the explanation of profit differences among firms. The analysis is wide-ranging, including a spatial framework where profits of two different firms are compared within the same period or a time framework by comparing profits of the same firm at two different dates or the general case where two different firms are compared in two different periods. In each case, the profit differential can be explained by several factors that we split into two groups, namely volume and price effects. On the volume side, we identify factors related to differences in management efficiency, in technology dominance and in size differences. On the price side, input and/or output price differences issuing from competitive advantages also explain a big part of the profit gap.

Among the volume effects the first two are related to the economic behavior of the firm[1]. A firm obviously makes more profit than another if it is more technically efficient in the sense that it uses the minimum of input in order to produce a given level of output or produce the maximum output given a level of input. Clearly, wastes are a source of inefficiency and of a lower profit. Second, a firm has to choose the best mix of inputs and outputs regarding the relative prices. A good allocation of resources is synonym of higher profit. The third effect is related to the production technology. Depending on the fact that they share the same technology, that one technology is more productive than the other or that the technologies share or not the same characteristics (returns to scale, input elasticity of substitution, output elasticity of transformation, marginal productivity of each input into each output, sub-additivity of the cost function…) profit gaps can occur. The last but not the least cause of profit differential is the size effect. This raises few comments. A firm could obviously make more profit if it is bigger. The second set of effects is based on prices. Clearly, differences in input prices or output prices directly impact the profit. Ideally, in a competitive market, firms are price taker and face the same prices for inputs and outputs. In the real world, price

[1] Obviously, we discuss each effect *ceteris paribus sic stantibus*. We therefore identify the marginal effect of dissimilarities between two firms.

variability comes from market imperfections which let room for a bargaining power between firms, suppliers and consumers and more generally between all stakeholders. That said, it is worth noting that we consider price effects exogenous from volume effects considered here. It is clearly an oversimplification since for example output price gaps coming from market power can be related to the size effect. It is however the traditional assumption in economics when dealing with production, profit or cost functions.

The decomposition of the profit differential can be conceived at various level of analysis. First we can deal with decomposition into a single volume effect and a single price effect. In this case, we don't need to introduce production technologies and only information on input/output price and quantity data is necessary. This decomposition has the advantage of simplicity but does not lead to a thorough understanding of the explicative factors of profit gaps. In order to refine the diagnosis, we need to introduce economic concepts such as production frontiers and efficiency measures so as to identify relevant benchmarks to which we can compare observed firms. In this framework we can first consider short term effects by ignoring for example the possibility of switching to a more productive technology or by ignoring long term effects of competition (zero long term profits). We may also want a deeper diagnosis at the cost of stronger assumptions by introducing for example optimal market shadow prices or constant returns to scale technologies to model most productive scale size. At the end of the day, there is no one optimal level of decomposition but a multiplicity depending on how we want to use the results, how relevant are the assumption we have to make or how are the material contingency such as the richness or the availability of the data

In the next section, we turn to the definitions of notations and the presentation of necessary economic concepts. We propose an illustration based on a single input / single output technology which allows discussing the profit decomposition problem in a very intuitive way. Third section introduces the Bennet index (Bennet, 1920) which is the necessary tool to decompose a money value into price and volume components. While traditional Laspeyres or Paasche type indices are well-known and extensively used, they still remain ratio type indices and we therefore prefer using Bennet indices which are relevant to linearly additive value measures such as profit, revenue and cost. We end this section by proposing a first decomposition in two global effects. Section 4 defines and computes each component of the profit decomposition. The resulting measures are then aggregated to conclude this bottom-up approach by showing that the sum of all components allows recovering the full profit differential between two firms. The next section shortly discusses the empirical implementation of the decomposition. A final section concludes.

2) Notations and definition of concepts

As we are interested in the comparison firm profit, first consider two firms A and B which produce an output vector Y from an input vector X. Now define $Q=(Y,-X)$ as the netput vector. Considering R as the output price vector and W as the input price vector, the netput price vector is written $P=(R,W)$. Profit of firms A and B are then define by:

$$\begin{aligned}\Pi_{Q_A}^{P_A} &= R_A Y_A - W_A X_A = P_A Q_A \\ \Pi_{Q_B}^{P_B} &= R_B Y_B - W_B X_B = P_B Q_B\end{aligned} \qquad (1)$$

We could have simply written the profit of A as Π_A but it will be useful to consider its profit under alternative price system or alternative quantity vector (in particular those of the compared firm). The above notation allows doing that without ambiguity. The profit difference between the two firms is then defined by:

$$\Pi_{Q_A}^{P_A} - \Pi_{Q_B}^{P_B} = \left[\ R_A Y_A - W_A X_A \ - \ R_B Y_B - W_B X_B \ \right] = P_A Q_A - P_B Q_B \qquad (2)$$

Decomposing and explaining $\Pi_{Q_A}^{P_A} - \Pi_{Q_B}^{P_B}$ will be the core objective of the paper.

In production economics, each observed firm can be defined as a production plan or a Decision Making Unit (DMU) to which we associate its vector of netputs and the corresponding price vector. For firm A the corresponding production is defined by Q_A, P_A. A first important remark is that while A faces prices P_A it does not mean that A maximize its profit by producing Q_A. In other words, A is not necessarily profit efficient. We also notice that prices P_A are neither necessarily efficient in the sense that they lead firm A to maximize its productivity as it is expected under competitive markets. In order to assess these potential inefficiencies we have to compare observed firms to efficient production plans which we need to define properly. The latter will be deemed as benchmarks for observed firms and are the answer to an optimal economic behavior such as technical efficiency, allocative efficiency, profit maximization or most productive scale size. In our framework, we identify three useful benchmarks so as to compare firm profit.

In the short term, the reference technology for a firm is its current technology, immediately available and defined under variable returns to scale (VRS). In the short run, any firm can adjust its production plan in order to improve the technical efficiency or the profit efficiency but we do not consider more drastic changes such as the split of the current production over smaller and more productive units or the adoption of a new and more efficient technology. The latter are typically medium or long run changes. In this context, the first identified benchmark for a firm is technically efficient that is it maximizes outputs given the inputs used by the firm. The first thing to do to improve profit is eliminating any waste. This first benchmark is obviously located on the frontier of the production set defined by all the technically efficient firms. Taking the firm A for illustration, the benchmark is defined by A^1 with an associated production plan Q_{A^1}, P_A located on the VRS frontier. The quantities of this benchmark are the result of an optimization problem that projects the firm A on the frontier along a chosen direction (input oriented, output oriented or any predefined direction). This optimization is clearly based on volumes and this results in a volume effect. Prices associated to the benchmark are those of the firm A.

The second benchmark relies on the short run profit maximization under the observed prices of firm A. The benchmark is defined by A^2 with an associated production plan Q_{A^2}, P_A for which quantities are also the result of an optimization problem: $\max_{\tilde{Q}} \Pi_{\tilde{Q}}^{P_A} : \tilde{Q}, P_A \in T_A^{VRS}(X,Y)$. This benchmark is the profit maximizing firm given the price system of A. It is obviously technically efficient and located on the VRS frontier. As for the first benchmark, we face a volume effect.

In the medium run, a firm can imagine to abandon its current technology so as to adopt a new and more productive one. Therefore we are considering the case of a benchmark located on another technology. Clearly, this benchmark will serve as the basis to measure technology dominance between two firms or technical progress in case of a temporal comparison. This production plan A^3 defined by (Q_{A^3}, P_A) is located on the VRS technology of the other firm, say firm B, and is the result of the following optimization problem: $\max_{\tilde{Q}} \Pi_{\tilde{Q}}^{P_A} : (\tilde{Q}, P_A) \in T_B^{VRS}(X,Y)$. Let us notice that we use the technology of firm B but prices are those of the firm A.

A last component in the profit decomposition arises from the direct comparison of the two optimal benchmarks of two firms A^3 and B^3. Since we here compare two different firms of two different sizes with different prices, this last effect encompasses both a volume and a price effect. A decomposition of this value effect into its volume and price components will leads respectively to the size inefficiency effect and the price effect.

To sum up, in addition to observed firms, we define three benchmarks in order to measure the whole decomposition of the profit differential between two firms. We identify four volume effects, namely technical, allocative, technology dominance and size effects and one global price effect. However it is important to notice that, what we are interested in is not the absolute value of these inefficiencies but the relative value compared to another firm that is the differential of efficiency for each component between the two firms. The latter will explain why a firm makes more profit than another. We formally define below each the presented concept and illustrate the decomposition in Figure 1.

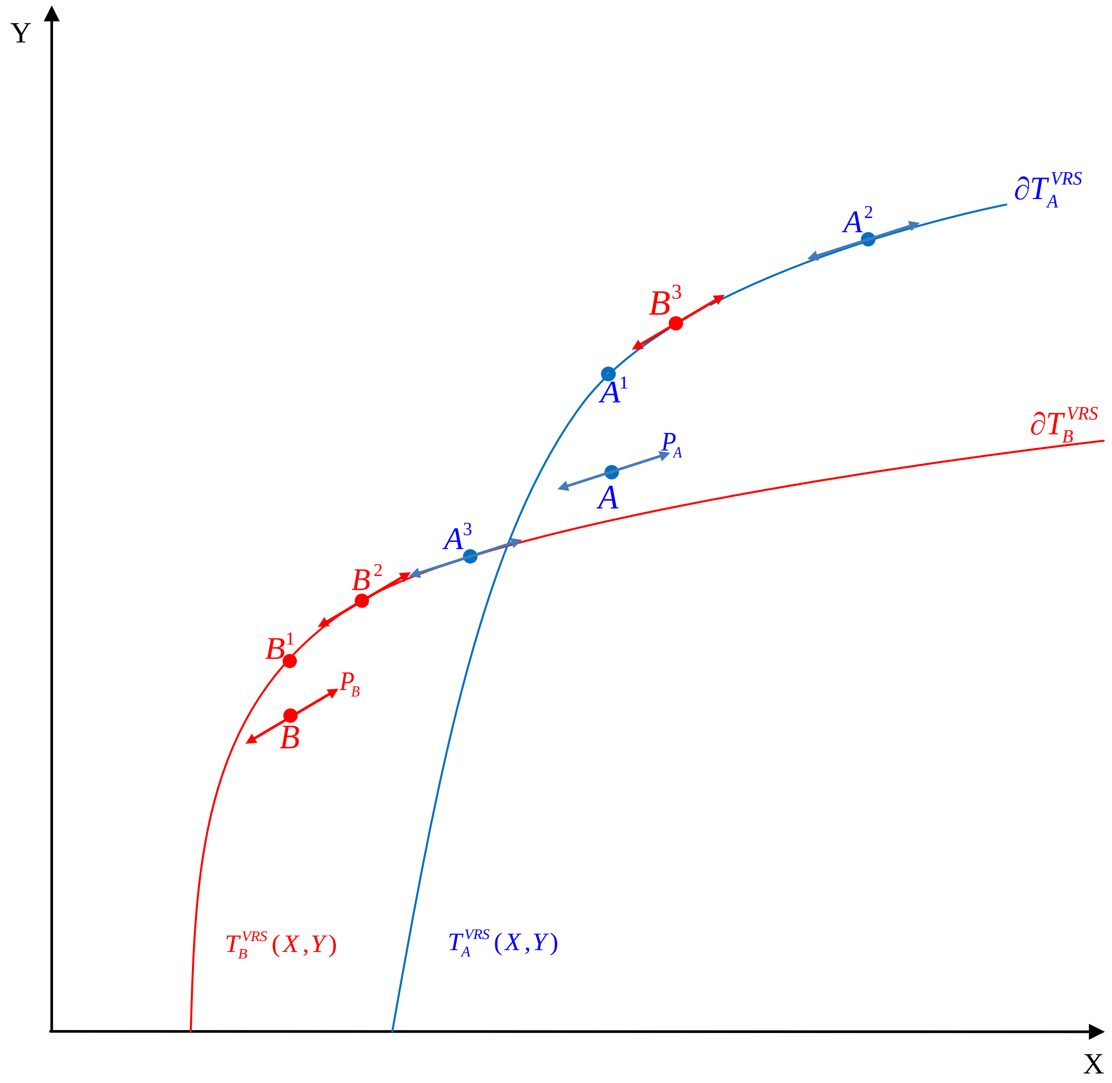
Definition 1:

Consider a sample of N observed firms. To each firm K $(K = 1,...,N)$ are associated:

1. A current production technology $T_K^{VRS}(X,Y)$ with the frontier denoted ∂T_K^{VRS} ;
2. An observed production plan (Q_K, P_K) ;

3. A short run benchmark K^1 which maximizes the technical efficiency, denoted Q_{K^1}, P_K and defined on the frontier $T_K^{VRS}(X,Y)$;
4. An optimal short run benchmark K^2 which maximizes the profit efficiency, denoted Q_{K^2}, P_K and defined on the frontier $T_K^{VRS}(X,Y)$;
5. A profit efficient benchmark K^3 defined on the technology of another firm K', denoted Q_{K^3}, P_K and defined on the frontier $T_{K'}^{VRS}(X,Y)$;

Figure 1: Production technologies and benchmarks related to each component of the profit decomposition



3) Bennet Indices as a tool to decompose the profit differential into price and volume components

Profit is money that is a value. Explaining profit differences is essentially working on values and each element of the decomposition must be measured in money. Traditional economic decomposition of a value measure relies on price and volume effects. Indices such as Laspeyres, Paasche or Fisher are very well-known and extensively used. However as stressed by Diewert (2005), dealing with profit means dealing with additive decomposition since the profit is defined as the difference between revenue and cost. Laspeyres, Paasche or Fisher indices are based on a multiplicative decomposition. We refer to Diewert (2005) for a thorough discussion on the properties and merits of each type of index in various economic contexts. For profit decomposition, Diewert unequivocally favors the Bennet index (Bennet, 1920) which appears as the most appropriate tool. We now define Bennet indices.

Definition 2:

a) The value Bennet index is defined as:

$$B_{A\ B}^{P_A P_B} = \Pi_{Q_A}^{P_A} - \Pi_{Q_B}^{P_B} \qquad (3)$$

b) The volume Bennet index is defined as:

$$B_{V A\ B}^{P_A P_B} = \left[\frac{1}{2}\ P_A + P_B \quad Q_A - Q_B\ \right] = \frac{1}{2}\left[\ \Pi_{Q_A}^{P_A} - \Pi_{Q_B}^{P_A}\ +\ \Pi_{Q_A}^{P_B} - \Pi_{Q_B}^{P_B}\ \right] \qquad (4)$$

c) The price Bennet index is defined as:

$$B_{P A\ B}^{P_A P_B} = \left[\frac{1}{2}\ Q_A + Q_B \quad P_A - P_B\ \right] = \frac{1}{2}\left[\ \Pi_{Q_A}^{P_A} - \Pi_{Q_A}^{P_B}\ +\ \Pi_{Q_B}^{P_A} - \Pi_{Q_B}^{P_B}\ \right] \qquad (5)$$

The interpretation of the volume Bennet index is immediate. The first term into the brackets measures the profit difference between firms A and B with the same price system, the one of A. The profit difference is then only due quantity differences. The second term does exactly the same with the price system of B. Finally, in order to not favor the one or the other price system, the arithmetic mean of these two effects is computed.

The interpretation of the price Bennet index is similarly immediate. The first term into the brackets measures the difference in the profit of firm A under the two price systems. The profit difference is then only due price differences since the quantities are unchanged. The second term does exactly the same for the firm B. Finally, in order to not favor the one or the other firm, the arithmetic mean of these two effects is computed.

Starting from the definitions of the Bennet indices, the profit differential between two firms admits the following decomposition:

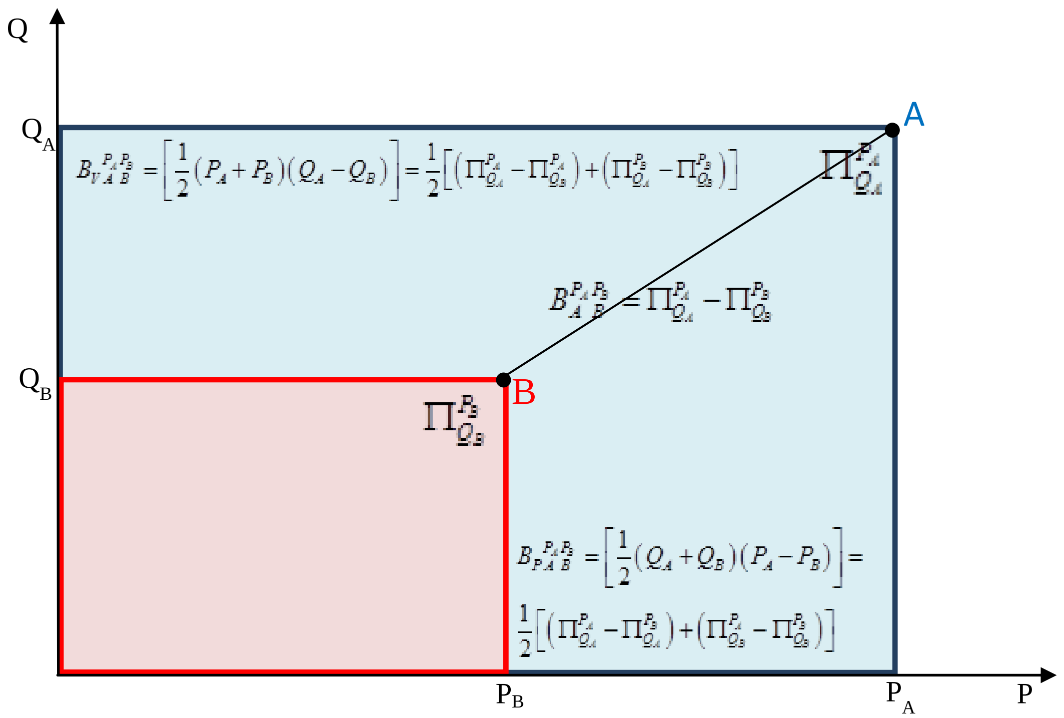
Property P1:

$$\Pi_{Q_A}^{P_A} - \Pi_{Q_B}^{P_B} = B_{A\,B}^{P_A P_B} = B_{V\,A\,B}^{\;P_A P_B} + B_{P\,A\,B}^{\;P_A P_B} \qquad (6)$$

Proof is relegated in Appendix.

Property P1 is the basis of the decomposition of profit into price and volume effects. A pictorial representation largely inspired by Diewert (2005) dreamily illustrates this decomposition.

Figure 2: Illustration of the decomposition of the Bennet Index



In Figure 2, two firms A and B are represented in the quantity/price space. The red rectangle defines the profit of firm B and the blue one defines profit of firm A. The profit difference between A and B is illustrated by the area between these two rectangles that is the visible blue part. Any decomposition of this area leads to a decomposition of the profit differences. The one proposed by Bennet is the following. Draw a segment between points A and B and compute the two related areas. The area attached to the quantity axis figures the volume effect and the area attached to the price axis the price effect. As we said above a lot of decomposition can be conceived but as shown by Diewert (2005) the one proposed by Bennet is probably the best and the most useful thanks to its many properties.

The Bennet decomposition as defined in (6): $\Pi_{Q_A}^{P_A} - \Pi_{Q_B}^{P_B} = B_{A\ B}^{P_A P_B} = B_{V A\ B}^{\ P_A P_B} + B_{P A\ B}^{\ P_A P_B}$ is the first step of our analysis which aims to explain profit gaps. The volume effect relies on the gap between quantities used by the two firms. However, this overall gap is explained by many factors: size differences, different technical choices, different ability to manage firm resources…. Some of these effects are short run, some others take place in a medium/long run. The same is true for the price effect. At optimum, prices should be the result of the input/output mix answering the consumers' demand with the best available technology. At the very best, they should induce zero profit for firms in long run equilibrium. If observed prices vary from firm to firm, opportunities emerge and firms can make a strictly positive profit in the short run. Price variations can be qualified as price advantages for some firms. We therefore retrieve market imperfections in the global price effect.

4) Identification of each component of the decomposition in a bottom-up approach

Clearly, our objective is to go one step beyond in the decomposition in order to further decompose the volume and price effect. However, this step can be done at the cost of more assumptions on production technologies and the way of identifying optimal benchmarks. We propose here to model each component individually and to propose an operational measure of each effect. We will see at the end whether our decomposition allows recovering or not the whole profit differential between two firms.

a. Identification and measure of the two profit efficiency effects

The first source of profit differential is the capacity of each firm to maximize its own profit. Consider again two firms A and B. Given their price system the profit maximizing production plans are respectively A^2 and B^2. To reach these benchmarks, each firm must first eliminating wastes (technical efficiency) and second choose the right mix of inputs and outputs regarding their prices (allocative efficiency). We therefore first compare the observed firms to their technically efficient benchmarks A^1 and B^1 and then to the profit efficient benchmarks A^2 and B^2. Remember that our objective is to measure the inefficiency gap between the two firms more than computing the absolute level of inefficiency for each individual firm. The latter can be a byproduct of our analysis. Formally, we have:

$$\begin{aligned}&\left[\Pi_{Q_A}^{P_A}-\Pi_{Q_{A^1}}^{P_A}+\Pi_{Q_{A^1}}^{P_A}-\Pi_{Q_{A^2}}^{P_A}\right]-\left[\Pi_{Q_B}^{P_B}-\Pi_{Q_{B^1}}^{P_B}+\Pi_{Q_{B^1}}^{P_B}-\Pi_{Q_{B^2}}^{P_B}\right]=\\&\left[B_{Q_A Q_{A^1}}^{P_A P_A}+B_{Q_{A^1} Q_{A^2}}^{P_A P_A}\right]-\left[B_{Q_B Q_{B^1}}^{P_B P_B}+B_{Q_{B^1} Q_{B^2}}^{P_B P_B}\right]=\left[B_{Q_A Q_{A^1}}^{P_A P_A}-B_{Q_B Q_{B^1}}^{P_B P_B}\right]+\left[B_{Q_{A^1} Q_{A^2}}^{P_A P_A}-B_{Q_{B^1} Q_{B^2}}^{P_B P_B}\right]\end{aligned} \quad (7)$$

The last term of the above equation is composed of value Bennet indices. However we can easily interpret these terms as volume effects since the benchmarks for each firm have the same price system as the considered firms. Therefore we can write:

$$\left[B_{Q_A Q_{A^1}}^{P_A P_A}-B_{Q_B Q_{B^1}}^{P_B P_B}\right]+\left[B_{Q_{A^1} Q_{A^2}}^{P_A P_A}-B_{Q_{B^1} Q_{B^2}}^{P_B P_B}\right]=\left[B_{V Q_A Q_{A^1}}^{P_A P_A}-B_{V Q_B Q_{B^1}}^{P_B P_B}\right]+\left[B_{V Q_{A^1} Q_{A^2}}^{P_A P_A}-B_{V Q_{B^1} Q_{B^2}}^{P_B P_B}\right] \quad (8)$$

This result follows directly the following property of Bennet indices.

Property P2:

For the same price system, the value Bennet index is equal to a volume Bennet index:

$B_{A\ B}^{P_A P_A}=B_{V A\ B}^{P_A P_A}$

Proof is relegated in Appendix.

In (7) or (8) we have the following interpretation of the two final terms:

- $\left[B_{Q_A Q_{A^1}}^{P_A P_A}-B_{Q_B Q_{B^1}}^{P_B P_B}\right]$ (9) is the « differential technical efficiency » effect ;

- $\left[B^{P_A \, P_A}_{Q_{A^1} Q_{A^2}} - B^{P_B \, P_B}_{Q_{B^1} Q_{B^2}} \right]$ (10) is the « differential allocative efficiency » effect.

These interpretations are quite natural in the sense that the technical efficiency is defined as the maximization of the outputs given a level of inputs. Technical efficiency is therefore a typical volume effect as we have verified above. On the other hand, the allocative efficiency is related to the choice of the best input/output mix given the observed price system. Again it is a volume effect choosing the right mix of quantities. Among all technical efficient production plan (on the frontier) only one is profit efficient given a price system. The distance to this optimal benchmark is measure by the « differential allocative efficiency » effect. The sum of these two effects can be interpreted as the « differential profit efficiency ».

b. Identification and measure of the technological dominance effect

A medium/long run component is related to the possibility of switching toward a new technology. When comparing two firms we can assume that they use different technologies. Within a homogenous industry we could have otherwise assumed that the technology is the same. The fact that firms operate under different technologies could lead to a dominance of one technology over the other and translates in a greater profit. In a time framework, we compare the technologies of the same firm over time and this effect clearly measures the technical progress. This effect must obviously be measured for a profit efficient production plan in order to isolate the marginal effect of technology dominance. Hence, we compare the profit efficient benchmark of a firm under its own production technology with the profir efficient benchmark of the same firm under the alternative technology:

$\Pi^{P_A}_{Q_{A^2}} - \Pi^{P_A}_{Q_{A^3}} - \Pi^{P_B}_{Q_{B^2}} - \Pi^{P_B}_{Q_{B^3}} = B^{P_A \, P_A}_{Q_{A^2} Q_{A^3}} - B^{P_B \, P_B}_{Q_{B^2} Q_{B^3}}$ (11) is the « differential of technological dominance » effect ;

This effect is easily interpreted. For firm A, the benchmark A_2 is profit maximizing on its own technology and A_3 would be the profit maximizing benchmark on the firm B technology keeping the price system of A. The profit difference between these two benchmarks only arises from the dominance of one technology over the other. As we have the same interpretation for firm B, we finally compute the differential of these effects.

One interesting question is the value of this effect if the two firms have the same technology. Obviously the answer must be zero since we have no dominance of a technology compared to itself. We verified easily this result. Actually if A and B share the same technology then $A_2 = A_3$ and $B_2 = B_3$ and finally: $B^{P_A\ P_A}_{Q_{A^2}\,Q_{A^3}} - B^{P_B\ P_B}_{Q_{B^2}\,Q_{B^3}} = B^{P_A\ P_A}_{Q_{A^2}\,Q_{A^2}} - B^{P_B\ P_B}_{Q_{B^2}\,Q_{B^2}} = 0$.

c. Identification and measure of the size effect and the price effect

It is clear that one of the main sources of profit differential between two firms is the size effect. By comparing a multinational company to a SME, we obviously expect more profit for the former. Another obvious source of differentiation is the price system of each firm. Competitive advantages often give better prices to a firm on both suppliers and clients sides. This clearly leads to higher profits.

One main difference emerges in our analysis at this level. While we have so far compared each firm to its own benchmark, we will compare directly the two firms here. In the short run, the optimal size is given by the profit maximizing production plan. We could have introduced a concept of optimal size in a medium/long term approach (as the concept of the Most Productive Scale Size, MPSS, defined on a constant returns to scale technology) but we prefer to stay in a short run analysis under the VRS technology. We therefore consider that the optimal size is given by the profit efficient benchmark and we only need to directly compare the size of this benchmark to the size of the other benchmark.

The same is true for the prices. It could have been possible to introduce a concept of optimal prices in the analysis based for example on long run assumptions of perfect competition but it will lead (at this stage) to strong and perhaps unrealistic assumptions. We therefore prefer to stick with the observed prices of each firm and compare one observed price system to another. These two effects are then defined as:

$$\Pi^{P_A}_{Q_{A^3}} - \Pi^{P_B}_{Q_{B^3}} = B^{P_A\ P_B}_{Q_{A^3}\,Q_{B^3}} = B_{V\,Q_{A^3}\,Q_{B^3}}^{\ P_A\ P_B} + B_{P\,Q_{A^3}\,Q_{B^3}}^{\ P_A\ P_B} \qquad (12)$$

Since we compare directly firm A and B, the price systems are different and the value Bennet index does not reduce to a single volume effect. Therefore we need to decompose the value index into its price and volume components. They are interpreted as:

- $B_{V}{}^{P_A\ P_B}_{Q_{A^3} Q_{B^3}}$ (13) is the « size differential » effect ;
- $B_{P}{}^{P_A\ P_B}_{Q_{A^3} Q_{B^3}}$ (14) is the « price advantages » effect.

The size effect is easily interpreted. By definition, it is a volume effect equal to:

$$B_{V}{}^{P_A\ P_B}_{Q_{A^3} Q_{B^3}} = \left[\frac{1}{2}\ P_A + P_B \quad Q_{A^3} - Q_{B^3}\ \right] = \frac{1}{2}\left[\ \Pi^{P_A}_{Q_{A^3}} - \Pi^{P_A}_{Q_{B^3}} \ + \ \Pi^{P_B}_{Q_{A^3}} - \Pi^{P_B}_{Q_{B^3}}\ \right] \qquad (15)$$

The first term of the bracket is the profit difference measured between A_3 et B_3 with the price system of A. Since prices are held constant, only quantities explain the profit differential. In the second term of the bracket, the same computation is done for firm B. We then average the two effects in order to not privilege the measure at firm A or B.

Along the same reasoning, the price effect is equal to:

$$B_{P}{}^{P_A\ P_B}_{Q_{A^3} Q_{B^3}} = \left[\frac{1}{2}\ Q_{A^3} + Q_{B^3} \quad P_A - P_B\ \right] = \frac{1}{2}\left[\ \Pi^{P_A}_{Q_{A^3}} - \Pi^{P_B}_{Q_{A^3}} \ + \ \Pi^{P_A}_{Q_{B^3}} - \Pi^{P_B}_{Q_{B^3}}\ \right] \qquad (16)$$

The first term of the bracket is the profit difference measured for firm A under the two price systems. Since quantities are held constant, only price differences explain the profit differential. In the second term of the bracket, the same computation is done for firm B. We then average the two effects.

d. ***Recovering the whole profit differential***

In the preceding subsections, we have proposed definitions and operational measures of each component for which we thought it could be a part of the difference of profit between two firms. Last but not least we need to prove that the sum of all these components effectively allows recovering the whole profit differential. The use of Bennet indices will prove very useful at this stage. As the main result of the paper, we state the following proposition.

Proposition 1

The profit differential between two firm A and B admits the following decomposition:

$$\Pi_{Q_A}^{P_A} - \Pi_{Q_B}^{P_B} = B_{Q_A Q_B}^{P_A P_B} = \left[B_{Q_A Q_{A^1}}^{P_A P_A} - B_{Q_B Q_{B^1}}^{P_B P_B} \right] + \left[B_{Q_{A^1} Q_{A^2}}^{P_A \; P_A} - B_{Q_{B^1} Q_{B^2}}^{P_B \; P_B} \right] + \left[B_{Q_{A^2} Q_{A^3}}^{P_A \; P_A} - B_{Q_{B^2} Q_{B^3}}^{P_B \; P_B} \right] + \left[B_{V\, Q_{A^3} Q_{B^3}}^{\;P_A \; P_B} \right] + \left[B_{P\, Q_{A^3} Q_{B^3}}^{\;P_A \; P_B} \right] \qquad (17)$$

where:

$\left[B_{Q_A Q_{A^1}}^{P_A P_A} - B_{Q_B Q_{B^1}}^{P_B P_B} \right]$ is the« technical efficiency differential» effect ;

$\left[B_{Q_{A^1} Q_{A^2}}^{P_A \; P_A} - B_{Q_{B^1} Q_{B^2}}^{P_B \; P_B} \right]$ is the « allocative efficiency differential» effect;

$\left[B_{Q_{A^2} Q_{A^3}}^{P_A \; P_A} - B_{Q_{B^2} Q_{B^3}}^{P_B \; P_B} \right]$ is the « technological dominance differential» effect ;

$B_{V\, Q_{A^3} Q_{B^3}}^{\;P_A \; P_B}$ is the « size differential » effect ;

and $B_{P\, Q_{A^3} Q_{B^3}}^{\;P_A \; P_B}$ is the « price advantages differential » effect.

Proof is relegated in Appendix.

5) Towards an empirical implementation

We end the paper by a short discussion on the empirical implementation. As we based our analysis on production sets and on benchmarks directly issued from the efficiency and productivity literature, we can readily use the theory and methodology developed in this field. All the theoretical background is based on Shephard's approach for modeling production technologies by production sets (Shephard, 1953, 1970; Färe and Primont, 1995). The technological frontier can be estimated either by parametric approaches such as Stochastic Frontier Analysis (SFA) or by non-parametric methods such as Data Envelopment Analysis (DEA). Benchmarks are identified through the optimization program exposed in the paper and can be readily computed through distance function and linear programs. All the methodological and computational material can be found in Färe, Grosskopf and Lovell (1994) among others.

Conclusion

In this paper, we develop a methodology to decompose and explain the profit differential between two firms. While it seems a rather simple question in economics and management a large range of factors might explain profit gaps. By considering Bennet indices as a key tool in the measure of profit differences, we split the many sources of dissimilarities in price and volume effects. We identify factors related to differences in management efficiency (technical and allocative), in productivity gaps (dominance of technology) in size differences and in competitive advantages resulting in price differentiation. The main result of the paper is to prove that a bottom up approach identifying and computing each effect allows recovering the whole observed profit differential between two firms.

In our analysis, short and medium run effects are privileged. We have only considered observed price of firm inputs and outputs. We therefore end with a global price effect. An extension of the model could be the introduction of a long run technology on which optimal prices are defined. It could be perfect competition prices i.e. the Law of One Price (loOP) for which a zero profit condition is assumed or shadow prices computed at optimal benchmarks on the technology. For instance, shadow prices can be estimated at the most productive scale size (MPSS). In the same way, questions relative to the optimal size of a firm could be introduced trough a constant returns to scale technology. However, these extensions would add stronger assumptions to the modeling.

Last but not least we propose a wide-ranging analysis which encompasses both a spatial framework for comparing firms within an industry and a time framework for explaining the sources of productivity growth of any firm. Beyond the theoretical interest of the decomposition, we trust that it can be a useful tool in a real life context. First, it can help the understanding of observed profit variability among firms. Second, it reveals strengths and weaknesses of firms, a higher profit driven by the size can hide waste or management inefficiency which minor the profit. Finally our decomposition provides the levers and drivers for action so as to improve the firm position among its competitors.

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Appendix

Property 1:

The value Bennet index is equal to the sum of the volume and price Bennet indices: $B_{A\ B}^{P_A P_B} = B_{V A\ B}^{\ P_A P_B} + B_{P A\ B}^{\ P_A P_B}$

Proof :

$$B_{V A\ B}^{\ P_A P_B} + B_{P A\ B}^{\ P_A P_B} = \frac{1}{2}\left[\ \Pi_{Q_A}^{P_A} - \Pi_{Q_B}^{P_A}\ +\ \Pi_{Q_A}^{P_B} - \Pi_{Q_B}^{P_B}\ \right] + \frac{1}{2}\left[\ \Pi_{Q_A}^{P_A} - \Pi_{Q_A}^{P_B}\ +\ \Pi_{Q_B}^{P_A} - \Pi_{Q_B}^{P_B}\ \right] =$$

$$\frac{1}{2}\ \Pi_{Q_A}^{P_A} - \Pi_{Q_B}^{P_B}\ + \frac{1}{2}\ \Pi_{Q_A}^{P_A} - \Pi_{Q_B}^{P_B}\ = \Pi_{Q_A}^{P_A} - \Pi_{Q_B}^{P_B} = B_{A\ B}^{P_A P_B}\ \square$$

Property 2:

a) If we held the prices constant, a value Bennet index is equal to a volume Bennet index:

$B_{A\ B}^{P_A P_A} = B_{V A\ B}^{\ P_A P_A}$

Proof :

$$B_{A\ B}^{P_A P_A} = \left[\frac{1}{2}\ P_A + P_A\quad Q_A - Q_B\ \right] + \left[\frac{1}{2}\ Q_A + Q_B\quad P_A - P_A\ \right] = \frac{1}{2}\ P_A + P_A\quad Q_A - Q_B\ = B_{V A\ B}^{\ P_A P_A}\ \square$$

b) If we held the quantities constant, a value Bennet index is equal to a price Bennet index:

$B_{A\ A}^{P_A P_B} = B_{P A\ A}^{\ P_A P_B}$

Proof :

$$B_{A\ A}^{P_A P_B} = \left[\frac{1}{2}\ Q_A - Q_A\quad P_A + P_B\ \right] + \left[\frac{1}{2}\ Q_A + Q_A\quad P_A - P_B\ \right] = \left[\frac{1}{2}\ Q_A + Q_A\quad P_A - P_B\ \right] = B_{P A\ A}^{\ P_A P_B}\ \square$$

Proposition 1:

The profit differential between two firm A and B admits the following decomposition:

$$\Pi_{Q_A}^{P_A} - \Pi_{Q_B}^{P_B}\ = B_{Q_A Q_B}^{P_A P_B} = \left[B_{Q_A Q_{A^1}}^{P_A P_A} - B_{Q_B Q_{B^1}}^{P_B P_B}\right] + \left[B_{Q_{A^1} Q_{A^2}}^{P_A\ P_A} - B_{Q_{B^1} Q_{B^2}}^{P_B\ P_B}\right] + \left[B_{Q_{A^2} Q_{A^3}}^{P_A\ P_A} - B_{Q_{B^2} Q_{B^3}}^{P_B\ P_B}\right] + \left[B_{V Q_{A^3} Q_{B^3}}^{\ P_A\ P_B}\right] + \left[B_{P Q_{A^3} Q_{B^3}}^{\ P_A\ P_B}\right]$$

Proof :

We first need the two following properties of Bennet indices.

Property 3:

Bennet indices have the following symmetry properties:

$$B_{A\ B}^{P_A P_B} = -B_{B\ A}^{P_B P_A}$$
$$B_{V A\ B}^{\ P_A P_B} = -B_{V B\ A}^{\ P_B P_A}$$
$$B_{P A\ B}^{\ P_A P_B} = -B_{P B\ A}^{\ P_B P_A}$$

Proof :

$$B_{A\ B}^{P_A P_B} = \ \Pi_{Q_A}^{P_A} - \Pi_{Q_B}^{P_B} \ \ = - \ \Pi_{Q_B}^{P_B} - \Pi_{Q_A}^{P_A} \ \ = -B_{B\ A}^{P_B P_A}$$
$$B_{V A\ B}^{\ P_A P_B} = \frac{1}{2}\left[\ \Pi_{Q_A}^{P_A} - \Pi_{Q_B}^{P_A} \ + \ \Pi_{Q_A}^{P_B} - \Pi_{Q_B}^{P_B} \ \right] = -\frac{1}{2}\left[\ \Pi_{Q_B}^{P_A} - \Pi_{Q_A}^{P_A} \ + \ \Pi_{Q_B}^{P_B} - \Pi_{Q_A}^{P_B} \ \right] = -B_{V B\ A}^{\ P_B P_A}$$
$$B_{P A\ B}^{\ P_A P_B} = \frac{1}{2}\left[\ \Pi_{Q_A}^{P_A} - \Pi_{Q_A}^{P_B} \ + \ \Pi_{Q_B}^{P_A} - \Pi_{Q_B}^{P_B} \ \right] = -\frac{1}{2}\left[\ \Pi_{Q_A}^{P_B} - \Pi_{Q_A}^{P_A} \ + \ \Pi_{Q_B}^{P_B} - \Pi_{Q_B}^{P_A} \ \right] = -B_{P B\ A}^{\ P_B P_A} \ \square$$

Property 4:

The Bennet index is transitive:

$$B_{A\ C}^{P_A P_C} = B_{A\ B}^{P_A P_B} + B_{B\ C}^{P_B P_C}$$

Proof :

$$B_{A\ B}^{P_A P_B} + B_{B\ C}^{P_B P_C} = \ \Pi_{Q_A}^{P_A} - \Pi_{Q_B}^{P_B} \ + \ \Pi_{Q_B}^{P_B} - \Pi_{Q_C}^{P_C} \ \ = \Pi_{Q_A}^{P_A} - \Pi_{Q_C}^{P_C} = B_{A\ C}^{P_A P_C} \ \square$$

We are now in position to prove Proposition 1. Starting from:

$$\left[B_{Q_A Q_{A^1}}^{P_A P_A} - B_{Q_B Q_{B^1}}^{P_B P_B}\right] + \left[B_{Q_{A^1} Q_{A^2}}^{P_A\ P_A} - B_{Q_{B^1} Q_{B^2}}^{P_B\ P_B}\right] + \left[B_{Q_{A^2} Q_{A^3}}^{P_A\ P_A} - B_{Q_{B^2} Q_{B^3}}^{P_B\ P_B}\right] + \left[B_{V Q_{A^3} Q_{B^3}}^{\ P_A\ P_B} + B_{P Q_{A^3} Q_{B^3}}^{\ P_A\ P_B}\right]$$

By using Property 3 and by reorganizing the terms, we can write:

$$\left[B_{Q_A Q_{A^1}}^{P_A P_A} + B_{Q_{A^1} Q_{A^2}}^{P_A\ P_A} + B_{Q_{A^2} Q_{A^3}}^{P_A\ P_A}\right] + \left[B_{V Q_{A^3} Q_{B^3}}^{\ P_A\ P_B} + B_{P Q_{A^3} Q_{B^3}}^{\ P_A\ P_B}\right] + \left[B_{Q_{B^3} Q_{B^2}}^{P_B\ P_B} + B_{Q_{B^2} Q_{B^1}}^{P_B\ P_B} + B_{Q_{B^1} Q_B}^{P_B\ P_B}\right]$$

By using Property 1 to aggregate the term in the middle bracket and by using Property 4 on transitivity, we finally obtain:

$$B_{Q_A Q_{A^1}}^{P_A P_A} + B_{Q_{A^1} Q_{A^2}}^{P_A\ P_A} + B_{Q_{A^2} Q_{A^3}}^{P_A\ P_A} + B_{Q_{A^3} Q_{B^3}}^{P_A\ P_B} + B_{Q_{B^3} Q_{B^2}}^{P_B\ P_B} + B_{Q_{B^2} Q_{B^1}}^{P_B\ P_B} + B_{Q_{B^1} Q_B}^{P_B\ P_B} = B_{Q_A Q_B}^{P_A P_B} = \ \Pi_{Q_A}^{P_A} - \Pi_{Q_B}^{P_B} \quad \square$$